

Math Contest Level 2 - March 4, 2016

Coastal Carolina University

1. A square ink pad has sides of length 1 cm. It is covered in blue ink and carefully placed in the middle of a piece of white paper. The square pad is then rotated  $180^\circ$  about one of its corners so that all of the pad remains in contact with the paper throughout the turn. The pad is then removed from the paper. What area of paper, in  $\text{cm}^2$ , is colored blue?  
a)  $\pi + 2$                       b)  $2\pi - 1$                       c) 4                      d)  $2\pi - 2$                       e)  $\pi + 1$
2. How many solutions are there to the equation  $m^2 - 4n^2 = -3$  where  $m$  and  $n$  are integers?  
a) 0                      b) 1                      c) 2                      d) 4                      e) 6
3. Find the exact value of  $\sin^4(75^\circ) + \cos^4(75^\circ)$ .  
a)  $5/8$                       b)  $7/8$                       c) 1                      d)  $9/8$                       e)  $\frac{1+\sqrt{3}}{2}$
4. Which of the numbers is prime?  
a) 131                      b) 133                      c) 138                      d) 141                      e) 143
5. Which of the following integers is closest to  $\log_2(2^{999} + 2^{1000})$ ?  
a) 999                      b) 1000                      c) 1001                      d) 1999                      e) 2000

6. 600 students are divided into three groups of equal size for lunch. Each group has lunch at a different time, and students are randomly assigned to one of the three groups. The probability that three friends—Elena, Farrah, and Steve—will be assigned to the same lunch group is approximately:

- a)  $\frac{1}{27}$                       b)  $\frac{1}{9}$                       c)  $\frac{1}{8}$                       d)  $\frac{1}{6}$                       e)  $\frac{1}{3}$

7. The largest integer  $k$  such that  $5^k$  divides  $3(10!) + 12(5!) + 4(7!)$  is

- a) 1                      b) 2                      c) 3                      d) 4                      e) 5

8. A function  $f$  is defined on the set of positive integers such that  $f(xy) = f(x) + f(y)$  for all  $x$  and  $y$ . If  $f(10) = 14$  and  $f(40) = 20$ , find the exact value of  $f(500)$ .

- a) 29                      b) 30                      c) 39                      d) 48                      e) 50

9. Find the sum of the solutions of the equation  $\log_4(3x - 2) = \log_8(x^3)$ .

- a) -1                      b) 0                      c) 1                      d) 2                      e) 3

10. Automated high-speed trains leave London headed for Paris every 20 minutes. When they reach Paris they turn around and head back to London. The commute takes 2 hours in each direction. One day, Billie and his father take the London train to Paris and back, without getting off. How many trains will they pass that were going in the opposite direction?

- a) 11                      b) 12                      c) 16                      d) 22                      e) 24

11. Which of the following cubic polynomials has the root  $x = \sqrt[3]{8 + \sqrt{56}} + \sqrt[3]{8 - \sqrt{56}}$  ?

- a)  $x^3 + 6x - 16$    b)  $x^3 - 6x + 16$    c)  $x^3 + 6x + 16$    d)  $x^3 - 6x - 16$    e)  $x^3 + 6x - 16$

12. The number  $2016!$  ends in how many zeros?

- a) 201                      b) 402                      c) 403                      d) 483                      e) 502

13. Circle  $C$  is inscribed in a square,  $S$  of side length 1. The smaller circle  $D$  is nestled in a corner of  $S$  in such a way that it is tangent to two sides of  $S$  and the circle  $C$ . Find the radius of circle  $D$ .

- a)  $\frac{3-2\sqrt{2}}{2}$                       b)  $\frac{\sqrt{2}-1}{1+2\sqrt{2}}$                       c)  $\frac{\sqrt{2}-1}{2}$                       d)  $\frac{\sqrt{2}+1}{8}$                       e)  $2 - \sqrt{2}$

14. In a Greek village of 5761 inhabitants, at least  $n$  of the residents MUST have the same initials (first name, last name). What is the largest possible value of  $n$ ? (Note that there are 24 letters in the Greek alphabet.)

- a) 11                      b) 12                      c) 13                      d) 14                      e) 15

15. The largest solution of  $x^6 - x^5 - x^4 - x^3 - x^2 - x - 1 = 0$  is in which interval?

- a)  $[0, 1)$                       b)  $[1, 2)$                       c)  $[2, 3)$                       d)  $[3, 4)$                       e)  $[4, \infty)$

16. Two fair six sided dice are rolled repeatedly. What is the probability that a sum of 5 will occur before a sum of 8?

- a)  $\frac{4}{9}$                       b)  $\frac{5}{11}$                       c)  $\frac{6}{13}$                       d)  $\frac{7}{15}$                       e) other

17. In a plane, points  $A$  and  $B$  are on the same side of line  $l$ . If  $A$  and  $B$  are both 3 inches from  $l$  and 4 inches apart, find the radius of the circle that is tangent to  $l$  passing thru  $A$  and  $B$ .

- a)  $13/6$  in                      b)  $7/3$  in                      c)  $15/6$  in                      d)  $8/3$  in                      e)  $17/6$  in



