

# Math Contest - Level 1

March 17, 2017

- 1) A total of \$450 was donated to charity by 25 employees. If 15 employees donated at least \$12 but less than \$19 and 9 employees donated at least \$19, what is the maximum amount in dollars, that the last employee could have donated?  
A. 70   B. 79   C. 89   D. 99   E. none of these.
  
- 2) A cylindrical can with a diameter of 14 cm and a height of 28 cm contains 2 spherical balls each with a diameter of 14 cm. What percentage of the space in the can is NOT occupied by the balls?  
A.  $33\frac{1}{3}\%$    B. 25%   C.  $16\frac{2}{3}\%$    D.  $66\frac{2}{3}\%$    E.  $8\frac{1}{3}\%$
  
- 3) Suppose  $f(x) = 5x - 4$ . Solve  $f(f(x)) = 76$  for  $x$ .  
A. 4   B. 5   C. 6   D. 76   E. 77
  
- 4) How many quarts of pure antifreeze must be added to 16 quarts of water to create a 20% solution? Round your answer to the nearest tenth.  
A. 10   B. 8   C. 4   D. 2   E. 3
  
- 5) Find the circumference of a circle that circumscribes a regular hexagon with an area of  $42\sqrt{3}$  cm<sup>2</sup>.  
A.  $4\pi\sqrt{7}$  cm   B.  $21\pi\sqrt{3}$  cm   C.  $2\pi\sqrt{42}$  cm   D.  $4\pi\sqrt{42}$  cm   E.  $6\pi\sqrt{42}$  cm

6) Architect Maria was given the job to organize a yearly architecture seminar. She was given a salary supplement of \$2017. She did such a great job that next year her supplement was increased by 300%. The next year the seminar bombed so her supplement was then decreased by 75%. What is her salary supplement now?

- A. \$1512.75   B. \$2017   C. \$4536   D. \$6051   E. \$0

7)  $|x - 2| < 8$  if and only if  $a < x + 9 < b$ . Find the values of  $a$  and  $b$ .

A.  $a = -3, b = 19$

B.  $a = 3, b = 19$

C.  $a = -6, b = 1$

D.  $a = 3, b = 1$

E.  $a = -3, b = 2$

8) Rey is 12 years younger than Sebastian. Five years ago, Rey was half Sebastian's age. How old will Sebastian be next year?

- A. 15   B. 20   C. 25   D. 30   E. 35

9) Find the area in square feet of a rectangle whose perimeter is 46 ft and diagonal is 17 ft.

- A. 60   B.  $60\sqrt{2}$    C. 120   D.  $120\sqrt{2}$    E. 240

10) The area of a certain equilateral triangle is equal to twice its perimeter. What is the length of one of its sides?

- A.  $2\sqrt{3}$    B.  $4\sqrt{3}$    C.  $6\sqrt{3}$    D.  $8\sqrt{3}$    E.  $10\sqrt{3}$

11) Simplify

$$\frac{1}{\sqrt{5}+\sqrt{2}} + \frac{1}{\sqrt{5}-\sqrt{2}}$$

- A.  $\frac{1}{2}$    B. 2   C.  $\frac{3\sqrt{5}}{2}$    D.  $\frac{\sqrt{10}}{2}$    E.  $\frac{2\sqrt{5}}{3}$

- 12) For all integer choices of  $n$  what value is  $(-1)^{4n^4+6n^2+1}$ ?
- A. 1   B.  $-1$    C.  $(-1)^n$    D.  $(-1)^{n^2+1}$    E.  $(-1)^{n^4+1}$
- 13) If  $x - \frac{1}{x} = 10$  then what is  $x^2 + \frac{1}{x^2}$ ?
- A. 100   B. 102   C. 104   D. 98   E. 96
- 14)  $a$ ,  $b$ , and  $c$  are three consecutive odd integers such that  $a < b < c$ . If  $a$  is halved to become  $m$ ,  $b$  is doubled to become  $n$ ,  $c$  is tripled to become  $p$ , and  $k = mnp$ , which of the following is equal to  $k$  in terms of  $a$ ?
- A.  $3a^3 + 18a^2 + 24a$
- B.  $3a^3 + 9a^2 + 6a$
- C.  $\frac{11}{2}a + 16$
- D.  $6a^2 + 36a + 24$
- E.  $a^3 + 6a^2 + 4a$
- 15) A circle of radius 25 units has a chord going through a point that is located 10 units from the center. What is the shortest possible length that chord could have?
- A. 25   B.  $\sqrt{525}$    C. 40   D.  $\sqrt{1050}$    E.  $\sqrt{2100}$
- 16) What is the area of the square whose circumscribed circle has radius 20?
- A. 80   B.  $400\sqrt{2}$    C. 800   D.  $800\sqrt{2}$    E. 400

- 17) Suppose  $x$  and  $y$  are two real numbers that satisfy  $xy = 7$  and  $x^2y + xy^2 + x + y = 72$ . Find  $x^2 + y^2$ .
- A. 96   B. 27   C. 144   D. 81   E. 67
- 18) Your school is raffling a \$450 bicycle. Tickets are sold at 75 cents a piece. How many tickets must be sold for the school to make a profit of \$75?
- A. 550   B. 600   C. 650   D. 700   E. 750.
- 19) Each term of a certain sequence is calculated by adding a particular constant to the previous term. The 2<sup>nd</sup> term of this sequence is 27 and the 5<sup>th</sup> term is 84. What is the first term of this sequence?
- A. 20   B. 15   C. 13   D. 12   E. 8.
- 20) Car  $A$  started driving north from point  $X$  traveling at a constant rate of 40 miles per hour. One hour later, Car  $B$  started driving north from point  $X$  at a constant rate of 30 miles per hour. Neither car changed direction of travel. If each car started with 8 gallons of fuel, which is consumed at a rate of 30 miles per gallon, how many miles apart were the two cars when car  $A$  ran out of fuel?
- A. 30   B. 60   C. 90   D. 120   E. 150
- 21) BurgerTown offers many options for customizing a burger. There are 3 types of proteins and 7 condiments: lettuce, tomatoes, pickles, onions, ketchup, mustard and special sauce. A burger must include exactly one protein but may include as many or as few condiments as the customer wants. How many different burgers are possible?
- A.  $8!$    B.  $(3)(7!)$    C.  $(3)(8!)$    D.  $(8)(2^7)$    E.  $(3)(2^7)$
- 22) The probability of rain is  $\frac{1}{6}$  for any given day next week. What is the probability that it will rain on both Monday and Tuesday but NOT on Wednesday?
- A.  $\frac{5}{216}$    B.  $\frac{1}{216}$    C.  $\frac{1}{18}$    D.  $\frac{7}{18}$    E.  $\frac{5}{9}$

23) In a large bin there are 118 hats. They come in two colors, red or black; and in two styles, with or without stripes. If there are 44 red hats, 64 hats with stripes, and 33 red hats with stripes, how many black hats without stripes are there?

- A. 85   B. 43   C. 23   D. 10   E. 41

24) If  $2x \neq y$  and  $5x \neq 4y$  then

$$\frac{\frac{5x-4y}{2x-y}}{\frac{3y}{y-2x}} + 5$$

when simplified equals

- A.  $\frac{1}{2}$    B.  $\frac{3}{2}$    C.  $\frac{5}{2}$    D.  $\frac{7}{2}$    E.  $\frac{9}{2}$

25) Suppose  $x$  and  $y$  are real numbers such that  $7^x = 16$  and  $7^y = 4$ . What is  $2^{\frac{1}{x-y}}$ ?

- A. 7   B.  $\frac{1}{7}$    C.  $\sqrt{7}$    D.  $\frac{1}{\sqrt{7}}$    E. 1