

41th ANNUAL DR. SUBHASH C. SAXENA MATH CONTEST

March 3, 2020

COASTAL CAROLINA UNIVERSITY

Notes and directions:

- Do not turn this page over until you are told to do so.
- Fill in the SCANTRON form according to your proctor's instructions.
- Calculators are not permitted on this test.
- You have fifty minutes to complete the test. If you finish early, you should leave quietly and proceed to Hicks Dining Hall for lunch.
- The test is yours to keep, so use any extra space for scratch work.

Good luck!

Math Contest - Level 1

March 3, 2020

1. If $x - (x - (x - (x - (x - 1)))) = 1$, then x is
A) -2 B) -1 C) 0 D) 1 E) 2
2. The sum of two numbers is 18 and the difference of their squares is 144. The difference of the two numbers is
A) 8 B) 10 C) 12 D) 14 E) none of these
3. The sum of three numbers is 17. The first is 2 times the second. The third is 5 more than the second. What is the value of the largest of the three numbers?
A) 3 B) 6 C) 8 D) 10 E) none of these
4. A publisher puts out a new edition of a problem book by moving a block of problems at the very end of the book to the very beginning. If the new #7 is the old #33 and the new #18 is the old #8, the total number of problems in this book is
A) 26 B) 33 C) 36 D) 46 E) 62
5. An isosceles triangle has two sides of length 10 and one of length 12. What is its area?
A) 36 B) 48 C) 60 D) 92 E) none of these

6. A number of $2 \times 3 \times 5$ bricks are arranged in three stacks. In the first stack, the horizontal side of each brick is 3×5 . In the second stack, each horizontal side is 2×5 . In the third stack, each horizontal side is 2×3 . The three stacks have exactly the same height. The smallest number of bricks in the three stacks combined is
- A) 10 B) 15 C) 29 D) 30 E) 31
7. The graph of $|x| + |y| = 1$ is
- A) a straight line
B) two straight lines
C) parabola
D) a square with sides parallel to the x -axis and y -axis
E) a square with vertices on the x -axis and y -axis
8. If 2^{x+1} is between 8 and 10, then 4^{x+2} is between
- A) 16 and 20 B) 32 and 40 C) 64 and 100 D) 128 and 200 E) none of these
9. Two students attempted to solve a quadratic equation $x^2 + bx + c = 0$. Although both students did the work correctly, one miscopied the middle term and obtained the solution set $\{2, 3\}$, while the other miscopied the constant term and obtained the solution set $\{2, 5\}$. What is the correct solution set?
- A) $\{2, 7\}$ B) $\{3, 5\}$ C) $\{1, 5\}$ D) $\{1, 6\}$ E) none of these

10. $ABCDE$ is a pentagon. F is a point on AE such that AF is of length 1 centimeter. FAB, FBC, FCD and FDE are all isosceles right triangles, with the right angles at A, B, C and D respectively. The area, in square centimeters, of $ABCDE$ is

- A) 5 B) $\frac{13}{2}$ C) $5\sqrt{2}$ D) $\frac{15}{2}$ E) none of these

11. Compute $3\sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{\dots}}}}}}}$.

- A) 9 B) 18 C) 27 D) 36 E) 81

12. A ring of 10 grams is 60% gold and 40% silver. A jeweler wants to melt it down, add 2 grams of silver and add enough gold to make it 70% gold. How many grams of gold should be added?

- A) 4 B) 5 C) 8 D) 9 E) 12

13. P is a point inside a convex quadrilateral $ABCD$ of area 168 such that $PA = 9, PB = PD = 12$ and $PC = 5$. The perimeter of the quadrilateral is

- A) 38 B) 56 C) 58 D) 60 E) none of these

14. Dan is $\frac{2}{3}$ of the way through his run. After running another $\frac{1}{2}$ mile, he is $\frac{3}{4}$ of the way through his run. What is the total number of miles in his run?

- A) 4 B) 6 C) 18 D) 36 E) none of these

15. If $\frac{2}{x} = \frac{y}{3} = \frac{x}{y}$, then what is the value of x ?

- A) -6 B) 6 C) $\sqrt{12}$ D) $3\sqrt{2}$ E) none of these

16. Determine the product of the roots of the equation

$$|x|(|x| - 5) = -6$$

- A) -6 B) 6 C) 18 D) 36 E) none of these

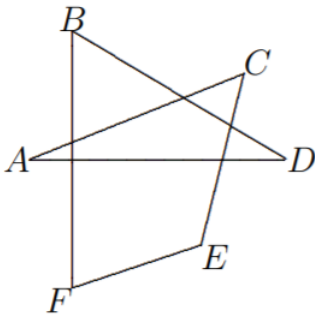
17. A pair of 6-sided dice are rolled, and the outcomes on each die differ by 1. Find the probability that the sum on this roll is 9.

- A) $\frac{1}{6}$ B) $\frac{1}{18}$ C) $\frac{1}{5}$ D) $\frac{1}{9}$ E) $\frac{3}{10}$

18. Alice, Brian, Colin, Debra and Ethel are in a hotel. Their rooms are on floors 1, 2, 3, 21 and 40 respectively. In order to minimize the total number of floors they have to cover to get together, the floor on which the get-together should be is

- A) 18 B) 19 C) 20 D) 21 E) none of these

19. In the diagram below, angle D equals 25 degrees. What is the sum of the angles at $A, B, C, D, E,$ and F ? Express your answer in degrees.



- A) 100° B) 155° C) 335° D) 360° E) 385°
20. Darren walks to the Diamond Music Shop at a rate of 6 km per hour and returns home, new violin in hand, at a rate of 5 km per hour. What was his average rate in km per hour for the total trip?
- A) $60/11$ B) 5.5 C) $2/11$ D) 5.75 E) other
21. There are three problems in a contest. Students win bronze, silver or gold medals if they solve 1, 2 or 3 problems respectively. Each problem is solved by exactly 60 students, and there are exactly 100 medalists. The difference between the number of bronze medalists and the number of gold medalists is

- A) 10 B) 20 C) 30 D) 40 E) not uniquely determined

22. How many integers between 10000 and 99999 have the property that all five digits are distinct and the last digit is the sum of the other four?

- A) 36 B) 90 C) 126 D) 154 E) none of these

23. What is the radius of the smallest circle that contains both of the circles $x^2 + y^2 = 1$ and $(x - 1)^2 + (y - 2)^2 = 4$?

- A) $(3 + \sqrt{2})/2$ B) $(3 + \sqrt{5})/2$ C) $(1 + \sqrt{5})/4$ D) $(3 + \sqrt{25})/4$ E) $3/2$

24. The positive integers from 1 to 999 are written in a row (with no spaces) in increasing order:

12345...998999

How many times does the sequence of characters "31" occur?

- A) 13 B) 19 C) 23 D) 29 E) 31

25. If $x > 0$, $x \neq 1$, and $(\log_2 x)^2 = \log_4 x$, then:

- A) $0 < x < 1$ B) $1 < x < 2$ C) $2 \leq x < 4$ D) $4 \leq x < \infty$
E) the situation is impossible