42^{nd} ANNUAL DR. SUBHASH C. SAXENA MATH CONTEST

March 3, 2023

COASTAL CAROLINA UNIVERSITY

Notes and directions:

- Do not turn this page over until you are told to do so.
- Fill in the SCANTRON form according to your proctor's instructions.
- Calculators are not permitted on this test.
- You have fifty minutes to complete the test. If you finish early, you should leave quietly and proceed to Hicks Dining Hall for lunch.
- The test is yours to keep, so use any extra space for scratch work.

Good luck!

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Math Contest - Level 1

March 3, 2023

1. If $\frac{2}{3} - \frac{1}{2} \div \frac{1}{6} + 3 = x$, which is the value of x?

   A. 4 B. $\frac{2}{3}$ C. 3 D. $-\frac{1}{19}$ E. 9

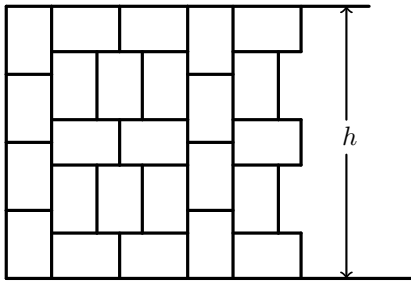
2. Suppose that a and b are two non-zero real numbers and

   $$a \cdot b = \frac{a}{b} = a - b.$$

   What is the value of $a + b$?

   A. $-\frac{3}{2}$ B. $-\frac{3}{4}$ C. 0 D. $\frac{1}{2}$ E. $-\frac{2}{3}$

3. The following figure shows a part of the tiling of a wall that is created by using identical rectangular bricks.

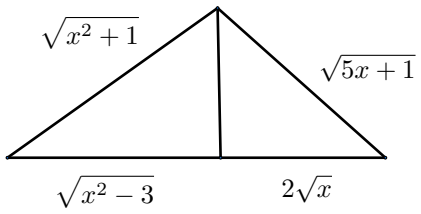


   Assuming that the length and width of the rectangles are measured by inches and have integer values, which one of the following could be the height h of the wall?

   A. 90 in B. 100 in C. 120 in D. 140 in E. 150 in

4. A triangle with its altitude is shown below. Find the value of x.

A. 2 B. 3 C. 4 D. 5 E. none of these



5. Suppose that $\frac{3}{5}$ of a container is filled with water. If we empty half of the water in the container the remaining water is 12 liters. How many liters of water can a full container carry?

A. 10 B. 20 C. 30 D. 40 E. 60

6. Simplify

$$\frac{0.1}{0.01}+\frac{0.01}{0.001}-\frac{0.001}{0.0001}$$

A. 0.1 B. 0.2 C. 10 D. 20 E. 100

7. Which of the following triplets are in between $\frac{1}{4}$ and $\frac{1}{3}$?

A. $\frac{6}{24}, \frac{7}{24}, \frac{8}{24}$ B. $\frac{10}{36}, \frac{11}{36}, \frac{12}{36}$ C. $\frac{13}{48}, \frac{14}{48}, \frac{15}{48}$ D. $\frac{17}{60}, \frac{18}{60}, \frac{20}{60}$ E. $\frac{18}{72}, \frac{19}{72}, \frac{20}{72}$

8. Given $x = \sqrt{2}$ and $y = \sqrt{3}$, write $\sqrt{2400}$ in terms of x and y.

A. $5xy$ B. $5x^3y$ C. $5x^5y$ D. $5x^2y^5$ E. $5x^3y^5$

9. Which one of the following statements is true?

A. $\sqrt{2} + \sqrt{7} = 3$ B. $\sqrt{\sqrt[3]{5}} = \sqrt[5]{5}$ C. $\sqrt{(1-\sqrt{3})^2} = 1 - \sqrt{3}$

D. $(1-\sqrt{3})^2 = 4 - 2\sqrt{3}$ E. $(\sqrt{11} - \sqrt{2})^2 = 9$

10. Suppose that a and b are real numbers and $a < 0 < 5 < b$. Which of the following is always correct?

A. $\frac{1}{a} < \frac{1}{b} < \frac{1}{5}$ B. $\frac{1}{b} < \frac{1}{5} < \frac{1}{a}$ C. $\frac{1}{a} < \frac{1}{5} < \frac{1}{b}$ D. $\frac{1}{5} < \frac{1}{a} < \frac{1}{b}$ E. $\frac{1}{b} < \frac{1}{a} < \frac{1}{5}$

11. What is the smallest integer x satisfying the following inequality

$$3x + 1 < 5x - 7?$$

A. 2 B. 3 C. 4 D. 5 E. 7

12. Suppose that x and y are two real numbers satisfying

$$-10 < x < 7,$$

$$2 < y < 5.$$

What is the greatest integer value of $x^2 + y^2$.

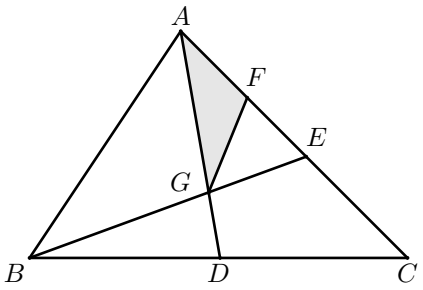
A. 84 B. 85 C. 120 D. 124 E. 135

13. Evaluate

$$\frac{30}{\sqrt{15}} + \frac{8\sqrt{3}}{3+\sqrt{5}}$$

A. $4\sqrt{15}$ B. $2\sqrt{5}$ C. $4\sqrt{5}$ D. $2\sqrt{3}$ E. $6\sqrt{3}$

14. In the following figure ABC is a triangle, AD and BE are medians, and F is the midpoint of AE. If the area of the shaded triangle ($\triangle AGF$) is 2 square inches, what is the area of $\triangle ABC$?



A. 6 in^2 B. 12 in^2 C. 24 in^2 D. 30 in^2 E. 48 in^2

15. One of Ken and Kelly lies on Mondays, Tuesdays and Wednesdays, and tells the truth on the other days of the week. The other lies on Thursdays, Fridays and Saturdays, and tells the truth on the other days of the week. At noon, the two had the following conversation:

Ken: I lie on Saturdays.

Kelly: I will lie tomorrow.

Ken: I lie on Sundays.

This conversation takes place on a

A. Monday B. Wednesday C. Thursday D. Saturday E. Sunday

16. An eight-inch pizza is cut into 3 equal sizes. A ten-inch pizza is cut into 4 equal sizes. A twelve-inch pizza is cut into 6 equal sizes. A fourteen-inch pizza is cut into 8 equal sizes. From which pizza would you take a slice if you want as much as possible?

A. 8-inch B. 10-inch C. 12-inch D. 14-inch E. does not matter

17. If $f(x) = x^x, f(f(x))$ is equal to

A. $x^{(x^{(x^x)})}$ B. $x^{(x^x)}$ C. $x^{(x^2)}$ D. $x^{(x^{(x+1)})}$ E. $x^{((x+1)^x)}$

18. Suppose Jamie can do a job in 4 hours and Janice in 5 hours. If they work together for 2 hours and then Jamie quits, how long will it take Janice to finish the job?

A. 0.25 hours B. 0.5 hours C. 0.75 D. 1 hour E. 1.25 hours

19. If $x + \dfrac{1}{x} = 3$, the value of $x^3 + \dfrac{1}{x^3}$ is

A. 21 B. 9 C. 12 D. 18 E. 27

20. If $r_1 = 1.46666\ldots$ and $r_2 = 0.407407407407\ldots$, find $\dfrac{r_1}{r_2}$.

A. $\dfrac{120}{37}$ B. $\dfrac{66}{185}$ C. $\dfrac{18}{5}$ D. $\dfrac{36}{11}$ E. $\dfrac{1466}{407}$

21. A clock has a minute hand which is 10 inches long. Then the area (in sq. inches) swept out by the minute hand between 10:33 AM and 10:38 AM on any given day is

A. $\dfrac{5\pi}{3}$ B. $\dfrac{25\pi}{12}$ C. $\dfrac{25\pi}{18}$ D. $\dfrac{25\pi}{3}$ E. $\dfrac{15\pi}{3}$

22. If $g(1) = 2$ and $g(n+1) = [g(n)]^2$, what is the value of $g(4)$?

A. 0 B. 4 C. 16 D. 64 E. 256

23. Determine the product of the roots of the equation

$$|x|(|x| - 7) = -12$$

A. 144 B. 128 C. 48 D. -128 E. -24

24. A 10 meters by $10\sqrt{2}$ meters rectangular building is surrounded by a large grassy field. A peg is pounded into the grass at a distance $5\sqrt{2}$ meters straight out from the midpoint of one of the long walls of the building. A goat is tied to the peg by a 20-meter rope. In square meters, the area of grass available to be eaten by the goat is

A. $\dfrac{325\pi}{2} + 25$ B. $250\pi + 25$ C. $300\pi + 50$ D. $325\pi + 50$ E. none of these

25. Suppose p and q are polynomials and

$$p(x + 3) = (x^2 + 2x + 1)q(x - 1) + x - 1 \, .$$

If the sum of the coefficients of p is 6, what is the remainder when $q(x + 1)$ is divided by $(x + 4)$?

A. 6 B. 7 C. 8 D. 9 E. 10