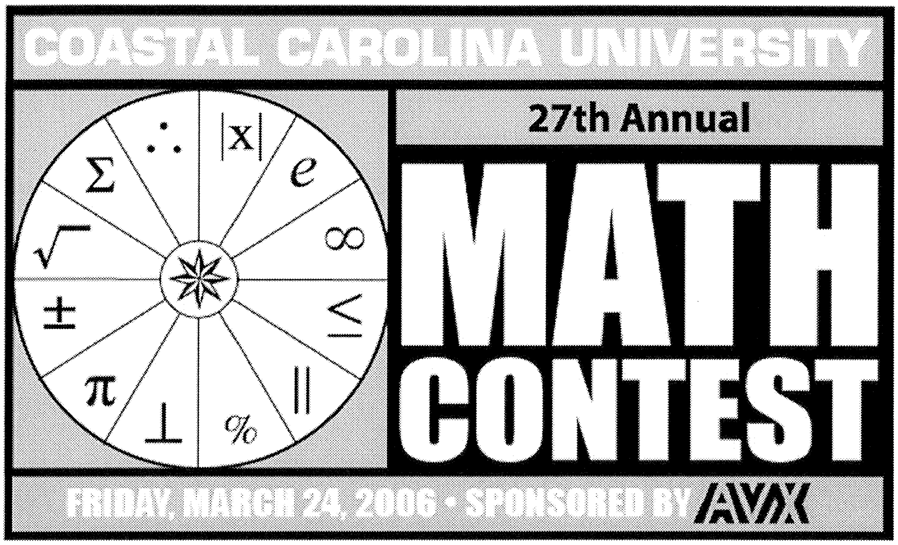


Math Contest
Level 2

Friday, March 24th, 2006

Notes And Directions:

- Do not turn this page over until you are told to do so.
- Fill in the SCANTRON form according to you proctor's instructions.
- Number 2 pencils should be used to bubble in the SCANTRON form. If you need a pencil, ask your proctor for one.
- Calculators are not permitted on this test.
- You have one hour to complete the test. If you finish early, you may leave quietly.
- You may keep your copy of the test. Scratch work can be done in the margins or on the back of the test pages.

COASTAL CAROLINA
UNIVERSITY

1. What is the integer n for which $5^n + 5^n + 5^n + 5^n + 5^n = 5^{25}$?

(a) 23 (b) 1 (c) 24 (d) 25 (e) None of the above

2. Let $N = 123456789$. If the digits 1 and 4 are interchanged in N, we obtain 423156789, which is an integral multiple of 11. Find all other integral multiples of 11 that can be obtained by interchanging two digits of N.

(a) 1 (b) 2 (c) 3 (d) 4 (e) more than 4

3. In how many ways can the letters A,B,C and D be arranged in a sequence so that A is not in position 3, B is not in position 1, C is not in position 2 and D is not in position 4?

(a) 6 (b) 3 (c) 9 (d) 12 (e) none of the above

4. A square has an area of R^2. An equilateral triangle has a perimeter of E. If r is the perimeter of the square and e is the side of the equilateral triangle, then, in terms of R and E, $e + r =$

(a) $\frac{E+R}{7}$ (b) $\frac{3E+4R}{3}$ (c) $\frac{E+12R}{3}$ (d) $\frac{12E+R}{3}$ (e) none of the above

5. The College of Hard Knox belongs to a six school league in which each school plays four games with each of the other schools. No tied games ever occur and the other five schools finished this season having won, respectively, 20%, 30%, 35%, 60%, and 80% of the league games they played. What was the College of Hard Knox's final winning record in the league this season (expressed as a percent)?

(a) 25% (b) 75% (c) 40% (d) 60% (e) none of the above

6. For $n = 1, 2, \ldots$, let $T_n = 1 + 2 + \ldots + n$. Which of the following statements is correct?

(a) There is no value of n for which T_n is a positive power of 2. (A positive power of two is an integer of the form 2^k where k is a positive integer.)

(b) There is exactly one value of n for which T_n is a positive power of 2.

(c) There are exactly two values of n for which T_n is a positive power of 2.

(d) There are more than two, but finitely many values of n for which T_n is a positive power of 2.

(e) There are infinitely many values of n for which T_n is a positive power of 2.

7. One hundred positive integers are arranged in a row. The sum of any six adjacent integers is the same as the sum of any other six adjacent integers. The value of the eleventh integer is 11, the value of the twenty-second integer is 22, the value of the thirty-third integer is 33, the value of the forty-fourth integer is 44, the value of the fifty-fifth integer is 55. The sum of the nineteenth and the ninety-ninth integer

(a) cannot be uniquely determined from the given information.

(b) must be less than 60.

(c) must be greater than or equal to 60 and must be less than 75.

(d) must be greater than or equal to 75 and must be less than 90.

(e) must be greater than 90.

8. If i represents the imaginary unit, what is the ordered pair of real numbers (a, b) for which $(1 + i)^{13} = a + bi$?

(a) (-2,-2) (b) (64,64) (c) (4,64) (d) (-64,-64) (e) none of the above

9. A box contains b red, $2b$ white and $3b$ blue balls, where b is a positive integer. Three balls are selected at random and without replacement from the box. Let $p(b)$ denote the probability that no two of the selected balls have the same color. Then,

(a) There is no value of b for which $p(b) = \frac{1}{6}$.

(b) There is exactly one value of b for which $p(b) = \frac{1}{6}$, and this value is less than 10.

(c) There is exactly one value of b for which $p(b) = \frac{1}{6}$, and this value is greater or equal to 10, but is less than 100.

(d) There is exactly one value of b for which $p(b) = \frac{1}{6}$, and this value is greater than 100.

(e) There is more than one value of b for which $p(b) = \frac{1}{6}$.

10. The positive integers $a_1, a_2, \ldots, a_{20}$ satisfy the following properties.

(a) $a_1 = 20$

(b) $a_{20} = 5$, and

(c) for $1 \leq i \leq 19$, $a_{i+1} - a_i \geq -1$.

(a) The value of a_{10} is uniquely determined.

(b) a_{10} can take on any one of 5 distinct values.

(c) a_{10} can take on any one of 9 distinct values.

(d) a_{10} can take on any one of 10 distinct values.

(e) a_{10} can take on any one of 15 distinct values.

11. Suppose that f is a function such that $f(3x) = \frac{1}{1+x}$ for $x > 0$. If $f(2002)$ is expressed in the form $\frac{p}{q}$ where p and q are positive integers having no common divisor other than 1, the value of $p + q$ is

(a) 2002 (b) 2005 (c) 2008 (d) 2011 (e) none of the above

12. A pair (M, N) of two digit positive integers, with $M \leq N$, is said to be *reversible* provided:

(a) if the digits of M are reversed, then the resulting integer is twice as large as N, and

(b) if the digits of N are reversed, then the resulting integer is twice as large as M.

Which of the following statements is correct?

(a) There are no reversible pairs (M, N) of two digit integers.

(b) There is exactly one reversible pair (M, N) of two digit integers.

(c) There are exactly two reversible pairs (M, N) of two digit integers.

(d) There are exactly three reversible pairs (M, N) of two digit integers.

(e) There are more than three reversible pairs (M, N) of two digit integers.

13. If b is a positive integer greater than 1, the base b expansion of a positive integer is denoted $(d_k d_{k-1} \ldots d_1 d_0)_b$ where the d_i are base b digits. Thus, the base 5 expansion of $(54)_{10}$ is $(204)_5$ since in base 10, $54 = 2(5^2) + 0(5^1) + 4(5^0)$. Similarly, the base 5 expansion of $(37)_{10}$ is $(122)_5$. How many of the integers between one and one thousand, inclusive, have a base 5 expansion that contains *at least* one zero?

(a) 1000 (b) 532 (c) 536 (d) 552 (e) none of the above

14. The diagram shows that an equiangular hexagon with side-length 6,7,9,10,11, and 14 can be inscribed in an equilateral triangle with side-length 30. This same equiangular hexagon can also be inscribed in an equilateral triangle with side-length $n \neq 30$. What is this value of n?

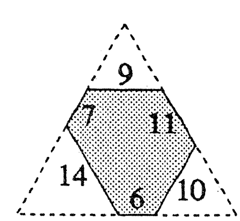
(a) 24

(b) 27

(c) 20

(d) 21

(e) none of the above



15. In a certain sequence, the first number is 1995. The second number equals the first number divided by 1 more than the first number. The third number equals the second number divided by 1 more than the second number. From then on, each number in the sequence equals the previous number divided by 1 more than the previous number. What is the 1995th number in this sequence?

(a) $\frac{1995}{3978031}$ (b) $\frac{1994}{3978031}$ (c) $\frac{3978031}{1995}$ (d) $\frac{3978031}{1994}$ (e) none of the above

16. When n is in $\{0, 1, 2, 3, \ldots\}$, the notation $n!$ (which is read as "n factorial") is defined by $0! = 1$, $1! = 1$, $2! = 2 * 1 = 2$, $3! = 3 * 2 * 1 = 6$ and $n! = 1 * 2 * 3 * 4 * \ldots * n$ for $n > 3$. Three solutions of the equation

$$m! * (m + 1)! = n!$$

are $(m, n) = (0, 0), (m, n) = (0, 1), and (m, n) = (1, 2)$. The fourth solution to this equation is

(a) (6,10) (b) (5,10) (c) (5,6) (d) (4,5) (e) none of the above

17. The number of ways to replace the dashes in

$$q = \_\ \_\ \_\ \_\ \_\ \_\ 4\ \_\ \_$$

by the digits 1,2,3,4,5,6,7,8,9 in some order is $8! = 40,320$. How many of these 40,320 ways results in a 9-digit number q that is an integral multiple of 25?

(a) 720 (b) 1440 (c) 5040 (d) 120 (e) none of the above

18. If n is a positive integer, define $d(n)$ to be the number of positive integer divisors of n. For example, $d(10) = 4$ since 10 is divisible by 1,2,5,10. The value of $d(d(288000))$ equals

(a) 10 (b) 12 (c) 14 (d) 8 (e) none of the above

19. A **binary day** has the property that its representation in the form $mm/dd/yy$ (where mm are the decimal digits that representing the month, dd are the decimal digits representing the day and yy are the last two decimal digits of the year) contains only 0's and 1's. For example, January 1, 2001 was a binary day since its representation in the above form was 01/01/01. How many binary days occur in any span of 100 years?

(a) 9 (b) 12 (c) 16 (d) 36 (e) none of the above

20. The number of arrangements of the letters AAAABBBC in which either the A's appear together in a block of four letters or the B's appear together in a block of three letters is

(a) 50 (b) 44 (c) 54 (d) 26 (e) none of the above

21. For all $x > 0$, $\left(\frac{\sqrt{x^3}}{\sqrt[3]{x^2}}\right)^{3/5} =$

(a) x^2 (b) x (c) $\sqrt{x}$ (d) $\sqrt[3]{x}$ (e) $\sqrt[4]{x}$

22. In the figure, O is the center of the circle and A,B and C are three points on the circle. Suppose that OA =AB=2 units and angle OAC is 10 degrees. The length of arc BC is

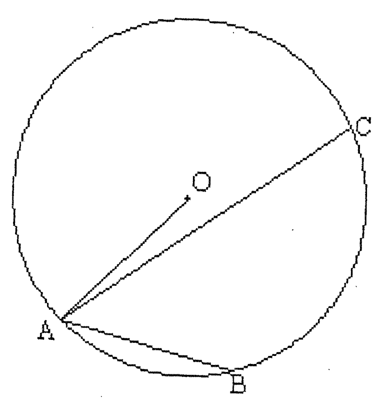
(a) $\frac{9\pi}{10}$

(b) $\frac{10\pi}{9}$

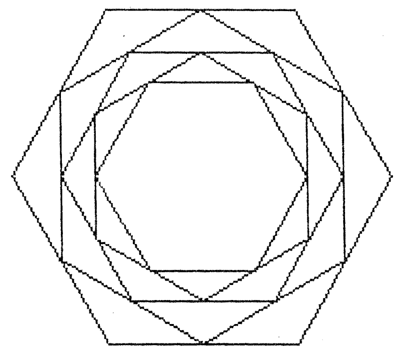
(c) 10π

(d) 9π

(e) none of the above



23. Let H_1 be a regular hexagon with sides of length 1 unit. Form hexagon H_2 by joining the midpoints of the sides of H_1. In a similar manner, construct hexagons $H_3, H_4, \ldots$. The first five hexagons are shown in the figure. Let A_n be the area in square units of hexagon H_n. Then, $\sum_{n=1}^{\infty} A_n$ equals



(a) $6\sqrt{3}$

(b) $\sqrt{3}$

(c) $3\sqrt{6}$

(d) 4

(e) none of the above

24. Suppose a and b are positive integers such that $(a+2b)(a-b) = 10$. What is the value of $2a - b$?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

25. If $\log_y x + \log_x y = 7$, then what is the value of $(\log_y x)^2 + (\log_x y)^2$?

(a) 40 (b) 43 (c) 45 (d) 47 (e) 49