Math Contest Level I – March 2007
Coastal Carolina University

(1) If $x + 2y = 5$, $z + 2x = 9$, and $y + 2z = 10$, then find the value of $x + y + z$.

(a) 2 (b) 4 (c) 6 (d) 8 (e) None of these

(2) In a rectangle we decrease one side by 3 and increase an adjacent side by 2 to form a square with area 25. What's the perimeter of the rectangle?

(a) 11 (b) 20 (c) 22 (d) 24 (e) Not enough info

(3) If the average (arithmetic mean) of 5 consecutive integers is 12, what is the sum of the least and greatest of the 5 integers?

(a) 24 (b) 14 (c) 12 (d) 11 (e) 10

(4) The number 0.01 is how many times as great as the number $(0.00001)^2$?

(a) 10^2 (b) 10^4 (c) 10^6 (d) 10^8 (e) 10^{10}

(5) A room is 30 feet square and 12 feet high. A jumping spider is located at one of the corners on the floor. An unsuspecting fly comes to rest at the diagonally opposite corner on the ceiling. If the fly stays put, what is the shortest distance the spider must crawl along the walls to catch the fly?

(a) $\sqrt{1944}$ (b) $\sqrt{2644}$ (c) $30 + \sqrt{1044}$ (d) $30 + \sqrt{1044}$ (e) 72

(6) What is the area of the region in the *xy*-plane defined by the inequalities $x \geq 0$, $y \geq x + 1$, and $2x + y \leq 10$.

(a) 10 (b) 12 (c) $\frac{27}{2}$ (d) $\frac{35}{2}$ (e) 30

(7) Seven black unit squares and 2 red ones on a table are to be assembled into a 3x3 pattern. How many different designs can be made? Two designs are different when they look different no matter how you rotate them on the table. Flipping is forbidden.

(a) 7 (b) 10 (c) 12 (d) 14 (e) 17

(8) The three vertices of a triangle are located at $(0,0)$, $(545,0)$, and $(751,915)$. The medians intersect at the point:

(a) $(434,304)$ (b) $(433,304)$ (c) $(433,305)$
(d) $(432,305)$ (e) None of these

(9) Two of six persons stole some apples. But who? A said "B and C." D said "E and F." E said "A and B." B said "F and D." C said "B and E." F cannot be found. Four of these had named one apple thief correctly and one incorrectly. The other had named both incorrectly. Who stole the apples?

(a) A, F (b) C, E (c) D, E (d) B, D (e) Not enough info

(10) For any two real numbers x and y, we define $x * y = 2(x + y - 1) - xy$. Which of the following is NOT true for all x, y, and z?

(a) $x * y = y * x$ (b) $x * (y * z) = (x * y) * z$ (c) $x * 1 = x$

(d) $x * 2 = 2$ (e) $x * \left(\dfrac{3 - 2x}{2 - x} \right) = 1$

(11) With a rational denominator, the expression $\dfrac{\sqrt{2}}{\sqrt{2} + \sqrt{3} - \sqrt{5}}$ is equivalent to:

(a) $\dfrac{3 + \sqrt{6} + \sqrt{15}}{6}$ (b) $\dfrac{\sqrt{6} - 2 + \sqrt{10}}{6}$ (c) $\dfrac{2 + \sqrt{6} + \sqrt{10}}{10}$

(d) $\dfrac{2 + \sqrt{6} - \sqrt{10}}{6}$ (e) None of these

(12) Let P be the increase in the circumference of a circle resulting from an increase of π units in the diameter. Then P equals:

(a) $\dfrac{1}{\pi}$ (b) π (c) $\dfrac{\pi^2}{2}$ (d) π^2 (e) 2π

(13) If the sum of all the angles except one in a convex polygon is 2190º, then the number of sides of the polygon must be:

(a) 13 (b) 15 (c) 17 (d) 19 (e) 21

(14) The function $f:\Re \to \Re$ satisfies the condition $m \cdot f(x-1) + n \cdot f(-x) = 2|x| + 1$. If $f(-2) = 5$ and $f(1) = 1$, then $m+n$ equals:

(a) 4 (b) 3 (c) $\dfrac{4}{3}$ (d) 6 (e) None of these

(15) The largest integer n so that 8^n divides 44^{44} is:

(a) 8 (b) 22 (c) 29 (d) 44 (e) 88

(16) Consider the first four rows of a triangular array:

$$\begin{array}{llll} 1 & & & \\ 2 & 3 & & \\ 4 & 5 & 6 & \\ 7 & 8 & 9 & 10 \end{array}$$

If this is extended indefinitely, then what is the sum of the 100^{th} row?

(a) 1000100 (b) 1000000 (c) 500000 (d) 500050 (e) 5000050

(17) A quadratic polynomial $p(x)$ satisfies $p(0) = 3$, $p(1) = 5$, and $p(2) = 8$. Then $p(5)$ equals:

(a) 22 (b) 23 (c) 24 (d) 25 (e) None of these

(18) If x is x% of y and y is y% of z, where x, y, and z are positive real numbers, then what is z?

(a) 100 (b) 200 (c) 10000 (d) Doesn't exist (e) Not enough info

(19) An x-by-y flag with $x < y$ consists of two perpendicular white stripes of equal width and four congruent blue rectangles at the corners. If the total area of the blue rectangles is half that of the flag, what is the length of the shorter side of each blue rectangle?

(a) $\dfrac{x - y + \sqrt{x^2 + y^2}}{4}$ (b) $\dfrac{x - y + \sqrt{x^2 + y^2}}{2}$ (c) $\dfrac{3x + y + \sqrt{x^2 + y^2}}{4}$

(d) $\dfrac{3x + y + \sqrt{x^2 + y^2}}{2}$ (e) None of these

(20) A game is played with a deck of ten cards numbered from 1 to 10. Shuffle the deck thoroughly, and then follow these two rules:

(i) Take the top card. If it's numbered 1, then you win. If not, then go to step (ii).

(ii) If this is the third time you have taken a card, you lose. Otherwise, put the card back into the k^{th} position from the top where k is the number on the card. Go back to step (i).

What is your probability of winning?

(a) $\dfrac{1}{5}$ (b) $\dfrac{5}{18}$ (c) $\dfrac{13}{45}$ (d) $\dfrac{3}{10}$ (e) None of these

(21) The sum $\frac{19}{99}+\frac{199}{999}+\frac{1999}{9999}+\ldots+\frac{199999999999}{999999999999}$ is closest to:

(a) $\frac{1}{9}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{5}{8}$ (e) $\frac{3}{4}$

(22) Let $f(x)=|x|-1$. For how many x-values is the function $f(f(f(f(x))))$ zero?

(a) 0 (b) 1 (c) 3 (d) 4 (e) 5

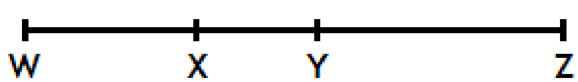
(23) Define $a \otimes b = \begin{cases} a+b, & a<b \\ ab, & a=b \\ a-b, & a>b \end{cases}$ Find $8\otimes((4\otimes 5)\otimes 1)$.

(a) 29 (b) 40 (c) 52 (d) 60 (e) 64

(24) A regular octagon is to be formed by cutting equal isosceles right triangles from each of the corners of a square. If the square has sides of one unit, then the legs of the right triangles have length:

(a) $\frac{2+\sqrt{2}}{3}$ (b) $\frac{2-\sqrt{2}}{3}$ (c) $\frac{2+\sqrt{2}}{2}$ (d) $\frac{1+\sqrt{2}}{3}$ (e) $\frac{2-\sqrt{2}}{2}$

(25) *Given:* The length of *WY* is 21. The length of *XZ* is 26. The length of *YZ* is twice that of *WX*. What is the length of *XY*?



(A) 5 (B) 10 (C) 11 (D) 16 (E) Cannot be determined