Math Contest Level 2 - March 2007

Coastal Carolina University

1. Dr. Incognito drives from home to work at an average speed of 45 mph. His average speed from work back to home is 50 mph. What is his average speed for the round trip from home to work and back?
   (a) 47.5 (b) $\frac{900}{19}$ (c) $\frac{900}{18}$ (d) $\frac{900}{17}$ (e) not determinable

2. The freshmen at Euler High School are required to take exactly 2 of the following 3 courses: Engish, Math, Social Studies. The freshman enrollments are: 20 in English, 17 in Math, 11 in Social Studies. How many freshman are there?
   (a) 24 (b) 25 (c) 26 (d) 27 (e) 48

3. A drawer contains 6 black socks and 6 white socks. If two socks are randomly selected from the drawer, what is the probability that the 2 socks will be the same color?
   (a) $\frac{5}{12}$ (b) $\frac{1}{2}$ (c) $\frac{2}{5}$ (d) $\frac{3}{7}$ (e) $\frac{5}{11}$

4. Ten people play a round-robin chess tournament (every person plays everyone else exactly once). In each game, the winner gets 3 points, the loser gets 0 points, and in case of a draw, each of the 2 players gets 1 point. If the total number of points given to all players is 130, how many games are draws?
   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

5. A certain ball, when dropped from any height, bounces back to $\frac{1}{3}$ of its original height. If the ball is dropped from 54 feet and continues to bounce up and down, what is the total distance (in feet) that the ball has travelled when it hits the ground for the fourth time?
   (a) 52 (b) 160 (c) 80 (d) 106 (e) 102

6. Given the standard coins issued by the U.S. treasury (1-cent, 5-cent, 10-cent, 25-cent, and 50-cent), what is the largest amount of money a person can have in coins without being able to make change for a dollar?
(a) \$0.99 (b)\$1.09 (c) \$1.14 (d) \$1.19 (e) \$1.24

7. Five test scores have a mean (average score) of 91, a median (middle score) of 92, and a mode (most frequent score) of 95. What is the sum of the two lowest test scores?
(a) 172 (b) 173 (c) 174 (d) 177 (e) 178

8. How many odd three digit numbers have three different digits?
(a) 60 (b) 288 (c) 300 (d) 320 (e) 360

9. Find the number of distinguishable permuations in the word Mississippi.
(a) 4^{11} (b) $\frac{11!}{4!7!}$ (c) $\frac{11!}{4!4!2!}$ (d) 11! (e) none of the above

10. What is the last digit of 517^{1003}
(a) 0 (b) 1 (c) 3 (d) 7 (e) 9

11. Which of the following rational numbers is equivalent to .083408340834...?
(a) $\frac{834}{9999}$ (b) $\frac{834}{10000}$ (c) $\frac{8}{90}$ (d) $\frac{417}{4995}$ (e) $\frac{417}{4950}$

12. The equation $x^2 + 2x + y^2 + 6y + 6 = 0$ defines a circle. What is the radius of the circle?
(a) 2 (b) $\sqrt{6}$ (c) $\sqrt{14}$ (d) 6 (e) none of the above

13. The equation $\sqrt{x+5} - \sqrt{x-2} + 1 = 0$ has
(a) 1 real root (b) 1 real root and 1 complex root
(c) 2 complex roots (d) 2 real roots (e) no roots

14. There is only one way to choose real numbers, M and N, so that when the polynomial, $5x^4 + 4x^3 + 3x^2 + Mx + N$ is divided by $x^2 + 1$, the remainder is 0. If M and N assume these unique values, what is $M - N$?
(a) -6 (b) -2 (c) 0 (d) 2 (e) 6

15. Suppose that $f(x+1, y) = f(x, y) + y + 1$, $f(x, 0) = x$, and $f(x, y) = f(y, x)$ for all real numbers x and y. What is $f(12, 5)$? (a) 5 (b) 60 (c) 12 (d) 17 (e) 77

16. Suppose that $\sin x + \cos x = a$. Which of the following is equivalent to $\sin^4 x + \cos^4 x$?
(a) a^4 (b) a^2 (c) $a^4 + a^2 + 1$ (d) $\frac{1+2a^2-a^4}{2}$ (e) $(\frac{a^2-1}{2})^2$

17. Find the exact value of $\cos(\frac{\pi}{12})$.
(a) $\frac{1}{2}$ (b) $-\frac{\sqrt{2+\sqrt{3}}}{2}$ (c) $.9659\ldots$ (d) $\frac{\sqrt{6}+\sqrt{2}}{4}$ (e) $\frac{\sqrt{3}}{2}$

18. Suppose that the focus of a parabola lies on the directrix. Then the parabola is
(a) an ellipse (b) a line (c) a circle
(d) a point (e) 2 parallel lines

19. Evaluate $(\frac{1}{2} + \frac{i}{2})^{10}$, where $i = \sqrt{-1}$.
(a) $\frac{1-i}{4}$ (b) $\frac{1}{8}$ (c) $\frac{-1+i}{16}$ (d) $\frac{i}{32}$ (e) none of the above

20. If a $4 \times 4 \times 4$ cube is made from 32 white unit cubes and 32 black unit cubes, what is the largest possible percentage of black surface area?
(a) 50 % (b) 60 % (c) 64 % (d) $66\frac{2}{3}$ % (e) 75 %

21. Circle C_1 with center P_1 and circle C_2 with center P_2 intersect in points Q and R such that $\angle QP_1R = 30°$ and $\angle QP_2R = 60°$. What is the ratio of the area of circle C_1 to the area of circle C_2?
(a) 2 (b) $\frac{1}{2-\sqrt{3}}$ (c) $7 + 4\sqrt{3}$ (d) $\frac{1}{4-\sqrt{2}}$ (e) $\sqrt{3}$

22. The line segments AB and CD are parallel and a distance of 4 apart. Suppose that AD intersects BC at a point P between the line segments. If $|AB| = 4$ (length of line segment) and $|CD| = 12$, how far is P from the line segment CD?
(a) 1 (b) 2 (c) 3 (d) 4 (e) 6

23. What is the area of the region common to two unit circle whose centers are $\sqrt{2}$ apart?
(a) $\frac{\pi}{2} - 1$ (b) $\frac{1}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{1}{2}(1 - \frac{\pi}{4}$ (e) $1 - \frac{\pi}{4}$

24. In triangle ABC, the point D lies on BC, and AD is the bisector of $\angle BAC$. If $|AB| = c$, $|AC| = b$, and $\angle CAD = w$, then what is $|AD|$?
(a) $\frac{bc \sin w}{b+c}$ (b) $\frac{b \cos w + c \sin w}{2}$ (c) $\frac{b(\sin 2w + \cos 2w)}{2}$ (d) $\frac{2bc \cos w}{b+c}$ (e) $\frac{2bc \sin w}{b+c}$

25. Let P be the point $(3, 2)$. Let Q be the reflection of P about the x-axis. Let R be the reflection of Q about the line $y = -x$, and let S be the reflection of R about the origin. What is the area of the convex quadrilateral $PQRS$?
(a) 14 (b) 15 (c) 16 (d) 17 (e) 18