Math Contest Level 2 - March 27, 2009

Coastal Carolina University

1. The number of seconds in 6 weeks equals $n!$. The value of n is:
   (a) 9 (b) 10 (c) 11 (d) 9! (e) 10!

2. The number of ways in which five A's and six B's can be arranged in a row which reads the same backwards and forwards is
   (a) 1 (b) 5 (c) 10 (d) 15 (e) none of these

3. Trains leave from Philadelphia for Harrisburg every hour on the hour. The trip takes three hours. Each train waits at the Harrisburg depot one half hour and then returns to Philadelphia. The number of trains going the other way that it will pass on its return trip is
   (a) 3 (b) 4 (c) 5 (d) 6 (e) 7

4. Superman and Yoda (working together) peel a bucket of potatoes in 20 minutes, Superman and Cinderella in 15 minutes, Cinderella and Yoda in 12 minutes. How long does it take Superman to peel a bucket of potatoes?
   (a) 1 hour (b) 45 minutes (c) 40 minutes (d) 30 minutes (e) none of the preceding

5. A and B are points on the circumference of a circle of radius r with center O. If the distance between A and B is r then the radian measure of the angle AOB is
   (a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) 1 (e) none of these

6. A country has three provinces, each province has three cities, each city has three wards and each ward has three electors. In a two-way election, a candidate wins a ward by getting more votes in the ward, wins a city by winning more wards in the city, wins a province by winning more cities in the province, and wins the election by winning more provinces. Only electors may vote, and they must vote. The minimum number of votes needed to guarantee winning the election is
(a) 16 (b) 40 (c)41 (d)66 (e) 81

7. The teacher asked, What is the largest possible diameter of a circular coin of negligible thickness which may be stored in a rectangular box with inner dimensions $7 \times 8 \times 9$? Ace said less than 8, Bea said 8, Ceci said strictly between 8 and 9, Dee said 9 and Eve said more than 9. Who was right?
(a) Ace (b) Bea (c) Ceci (d) Dee (e) Eve

8. What is the binary representation of the fraction 1/5?
(a) $.\overline{1001}$ (b) $.\overline{0011}$ (c) $.\overline{0101}$ (d) $.\overline{00111}$ (e) $.\overline{01101}$

9. An 8 inch chord is twice as far from the center of a circle as a 10 inch chord. The circumference of the circle in inches is
(a) $2\sqrt{7}\pi$ (b) $6\sqrt{7}\pi$ (c) $8\sqrt{7}\pi$ (d) $4\sqrt{7}\pi$ (e) $5\sqrt{7}\pi$

10. If $4^x - 4^{x-1} = 24$, then the value of $(2x)^x$ is
(a) $4\sqrt{2}$ (b) $12\sqrt{2}$ (c) $10\sqrt{5}$ (d) $4\sqrt{10}$ (e) $25\sqrt{5}$

11. Suppose that x and y are positive numbers for which $\log_9 x = \log_{12} y = \log_{16}(x+y)$. Then the value of $2y/x$ is
(a) $1+\sqrt{5}$ (b) $-1+\sqrt{5}$ (c) $1+\sqrt{3}$ (d) $-1+\sqrt{3}$ (e) $1-\sqrt{5}$

12. What are the last two digits of 2009^{2009}?
(a) 81 (b) 41 (c) 29 (d) 49 (e) 89

13. If $(1+i)^{100}$ is expanded and written in the form $a+bi$ where a and b are real numbers, then $a =$
(a) -2^{50} (b) $2^{50} - 100!/50!50!$ (c) $100!/(25!)^2 50!$ (d) $100!(\frac{-1}{50!50!} - \frac{1}{25!75!})$ (e) 0

14. What is the coefficient of x^3 in the expansion of $(1+x+x^2+x^3+x^4+x^5)^6$?
(a) 40 (b) 48 (c) 56 (d) 62 (e) 64

15. Arcsin(1/3) + Arccos(1/3) + Arctan(1/3) + Arccot(1/3) =
(a) π (b) $\pi/2$ (c) $\pi/3$ (d) $2\pi/3$ (e) $3\pi/4$

16. Three cards are in an envelope. One is green on both sides, another is red on both sides and the last is green on one side and red on the other. You select one card at random and you see that one side of the card is green. What is the probability that the other side is also green?
(a) 1/2 (b) 2/3 (c) 1/3 (d) 3/4 (e) 1/4

17. Let [x] represent the greatest integer that is less than or equal to x . Fox example, $[2.769] = 2$ and $[\pi] = 3$. Then what is the value of $[\log_2 2] + [\log_2 3] + [\log_2 4] + \ldots + [\log_2 99] + [\log_2 100]$?
(a) 480 (b) 481 (c) 482 (d) 483 (e) 484

18. There are 5! five-digit numbers that can be formed by permuting 1, 2, 3, 4, and 5, such as 12345, 12354, 21435, ..., 54321. The sum of all these numbers is
(a) 3,999,960 (b) 2,876,540 (c) 4,969,960 (d) 5,600,610 (e) 6,975,640

19. If $x^{\log_3 2} = 81$, then $x^{(\log_3 2)^2} =$
(a) 2 (b) 4 (c) 8 (d) 12 (e) none of these

20. In triangle ABC, the point D lies on BC, and AD is the bisector of angle BAC. If $|AB| = c$, $|AC| = b$, and angle $CAD = w$, then $|AD|$ is
(a) $(bc \sin w)/(b + c)$ (b) $(b \cos w + c \sin w)/2$ (c) $(b \sin 2w + c \cos 2w)/2$
(d) $(2bc \cos w)/(b + c)$ (e) $(2bc \sin w)/(b + c)$

21. Statistics for road use in a certain county show that in the past year, there were 32 accidents per 100,000 miles driven on rural roads and 18 accidents per 100,000 miles driven on city roads. Combined statistics for both rural and city roads show that there were 24 accidents per 100,000 miles driven. Let x be the total number of accidents on rural roads and y be the total number of accidents on city roads. The value of x/y is
(a) 3/2 (b) 4/3 (c) 5/4 (d) 5/3 (e) 7/5

22. The equation $x^{x\sqrt{x}} = (x\sqrt{x})^x$ has two positive solutions. One obvious solution is $x = 1$. The other one is $x =$
(a) 1/2 (b) 2/3 (c) 9/4 (d) 3/2 (e) none of these

23. When $x^{100} - 2x^{99} + 4$ is divided by $x^2 - 3x + 2$, the remainder is
(a) $x + 2$ (b) $x + 1$ (c) $2x + 1$ (d) $x - 1$ (e) $3x - 2$

24. Simplify $\dfrac{\cos(5x) + \cos(3x)}{\sin(5x) - \sin(3x)}$

(a) $\tan(x)$ (b)$\cot(x)$ (c)$\tan(2x)$ (d) $\cot(2x)$ (e) $\tan(2x)/\tan(x)$

25. Let $p(x)$ be the cubic polynomial $7x^3 - 4x^2 + K$. If the three roots of $p(x)$ form an arithmetic progression, then the value of K is
(a) 4/21 (b) 16/147 (c) 16/441 (d) 128/1323 (e) cannot be determined