

Math Contest Level 1 - March 11th, 2011

Coastal Carolina University

1. Let Adam, Bob, Carl, Dave and Eric form a line at a concession stand, and Carl and Dave refuse to stand next to each other. How many different lines are possible?
(a) 24 (b) 72 (c) 84 (d) 90 (e) 120

2. If $x^{1/2} + x^{-1/2} = 8$, then the value of $\frac{x^2 + 1}{x}$ is
(a) 63 (b) 64 (c) 65^x (d) 66 (e) none of above

3. If $N = \frac{\sqrt{\sqrt{5} + 2} + \sqrt{\sqrt{5} - 2}}{\sqrt{\sqrt{5} + 1}}$, then $N = ?$
a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 4 (e) none of above

4. The last digit of $8^{2010} + 9^{2011}$ is
(a) 1 (b) 3 (c) 5 (d) 7 (e) 9

5. Let $f(x) = \frac{x}{\sqrt{1+x^2}}$ and define $f^{(2)}(x) = f(f(x))$, $f^{(3)}(x) = f(f(f(x)))$, then $f^{(99)}(1) =$
(a) $\frac{1}{4}$ (b) $\frac{1}{\sqrt{99}}$ (c) $\frac{1}{9}$ (d) $\frac{1}{10}$ (e) none of above

6. If real numbers x and y satisfy $x^2 + y^2 - 4x - 2y + 5 = 0$, then $\frac{\sqrt{x} + y}{\sqrt{3y - 2\sqrt{x}}}$ is?
 (a) 1 (b) $3/2 + \sqrt{2}$ (c) $3 + 2\sqrt{2}$ (d) $3 - 2\sqrt{2}$ (e) none of above
7. Suppose that A, B, C, D are the vertices of a quadrilateral which lie on a circle, \overline{AD} intersects \overline{BC} at E , \overline{AB} intersects \overline{DC} at F , and $\angle BAD = 61^\circ$. Find $\angle AEB + \angle AFD$.
 (a) 58 (b) 60 (c) 61 (d) 65 (e) 75
8. For how many different values of k are both $\sqrt{k - 11}$ and $\sqrt{k + 64}$ integers?
 (a) 0 (b) 1 (c) 2 (d) 3 (e) none of these
9. It is possible to rearrange the numbers one through twelve on a clock so that the sum of any two adjacent numbers is prime. In order to perform this process, at least how many of the numbers must be relocated?
 (a) 3 (b) 4 (c) 5 (d) 6 (e) none of these
10. The sides of an equilateral triangle are parallel to three of the sides of a trapezoid whose vertices lie on the circle which circumscribes the equilateral triangle and whose base is the diameter of the circle. Find the ratio of the area of the triangle to the area of the trapezoid.
 (a) 1 (b) $\frac{\sqrt{3}+1}{3}$ (c) $\frac{4-\sqrt{3}}{2}$ (d) $3 - \sqrt{3}$ (e) none of these

11. Suppose that A, B, C are the vertices of a triangle such that $|\overline{AB}| = 6$, $|\overline{BC}| = 8$, and $|\overline{AC}| = 10$. Two identical circles are tangent to each other where one of the circles is tangent to both \overline{AB} and \overline{AC} while the other is tangent to both \overline{BC} and \overline{AC} . Which of the following numbers is the common diameter of each circle?
(a) $2\sqrt{3}$ (b) $\frac{20}{7}$ (c) $\frac{12}{5}$ (d) 3 (e) none of these
12. Find the sum of the first 100 terms of the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \dots$
(a) 755 (b) 845 (c) 927 (d) 945 (e) none of these
13. Suppose that a_0, a_1, a_2, \dots is a sequence of positive integers such that a_0 is not prime, and a_i is the number of positive divisors of a_{i-1} . For how many possible values of a_0 will none of the terms in the sequence be a perfect square?
(a) 0 (b) 1 (c) 2 (d) infinitely many (e) none of these
14. Which of the following numbers can not be obtained by inserting plus signs between the digits of the number 123456789. For example $12+3+456+7+89=567$. So, 567 can be obtained using this procedure.
(a) 144 (b) 153 (c) 189 (d) 375 (e) 486
15. N different numbers are placed on a circle so that every number is the product of its two adjacent numbers. Find the largest possible value of N .
(a) 4 (b) 6 (c) 8 (d) 10 (e) none of these

16. How many possible sums can be obtained using five distinct two digit numbers?
 (a) 424 (b) 426 (c) 428 (d) 430 (e) 432
17. If $\frac{b+2c-a}{2bc} + \frac{a+2c-b}{2ac} = \frac{a+b-2c}{ab}$, find the value of $\frac{a^2+b^2+c^2}{10c^2+4ab}$.
 (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) 2 (e) none of these
18. Suppose that $N \leq 1000$. For how many positive integer values of N is $N^2 + 8N - 85$ divisible by 101?
 (a) 0 (b) 2 (c) 6 (d) 9 (e) none of these
19. Consider a unit square with vertices A, B, C, D . Pick points A', B', C', D' on the sides $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$ respectively so that $|AA'| = |BB'| = |CC'| = |DD'| = \frac{1}{3}$. Find the area of the square enclosed by $\overline{AC'}, \overline{A'C}, \overline{BD'}, \overline{B'D}$.
 (a) $\frac{1}{10}$ (b) $\frac{1}{11}$ (c) $\frac{1}{12}$ (d) $\frac{1}{13}$ (e) $\frac{1}{14}$
20. Suppose that A, B, C are the vertices of a triangle where $|\overline{AB}| = |\overline{AC}|$ and $\angle A = 50^\circ$. Let D be the midpoint of \overline{BC} . Pick M on \overline{AD} and N on \overline{AC} so that $|\overline{MN}| = |\overline{MB}|$. Find $\angle MBN$ (in degrees).
 (a) 15 (b) 20 (c) 25 (d) 30 (e) 35

21. Suppose that x, y, z are real numbers satisfying $x^2 - 2|x| = y$, $y^2 - 2|y| = z$, and $z^2 - 2|z| = x$. Find the smallest possible value of $x + y + z$.
(a) -5 (b) -4 (c) 0 (d) 1 (e) none of these
22. How many ways can three numbers be chosen from the set $\{1, 2, \dots, 20\}$ so that their product is divisible by 4.
(a) 120 (b) 455 (c) 780 (d) 795 (e) 870
23. How many sequences of 5 positive integers (a, b, c, d, e) which will satisfy the following inequality $abcde \leq a + b + c + d + e \leq 10$?
(a) 26 (b) 32 (c) 102 (d) 116 (e) 120
24. What are the last two digits of 103^{4205} ?
(a) 01 (b) 23 (c) 43 (d) 63 (e) none of these
25. If $B_n = \frac{1}{2}(a^n + b^n)$ where $a = 3 + 2\sqrt{2}$ and $b = 3 - 2\sqrt{2}$, $n = 0, 1, 2, \dots$, then B_{12345} is an integer. The last digit of B_{12345} is
(a) 1 (b) 3 (c) 5 (d) 7 (e) 9