COASTAL CAROLINA UNIVERSITY

March 8, 2013

The 34th Annual Dr. Subhash Saxena Math Contest

LEVEL 1I

Notes and Directions:

- Do not turn this page until you are told to do so.
- Fill in the SCANTRON form according to your proctor's instructions. Make sure you put your name and your school's name at the top.
- Calculators are not permitted on this test.
- You have 50 minutes to complete the test. If you finish early, leave the classroom quietly and proceed to the Hicks Dining Hall for lunch.
- The test is yours to keep. Use any extra space for scratch work.

Math Contest Level 2 - March 8, 2013

Coastal Carolina University

1. The sum of the interior angles of a certain polygon is 2520. How many sides does it have?

   a) 12 b) 14 c) 16 d) 28 e) 42

2. Let $A = 2013 + \dfrac{1}{2013}$, $B = 2013 + \dfrac{1}{2013 + \dfrac{1}{2013}}$ and

$$C = 2013 + \frac{1}{2013 + \dfrac{1}{2013 + \dfrac{1}{2013}}}.$$

   a) $A < B < C$ b) $A < C < B$ c) $B < A < C$ d) $B < C < A$ e) $C < B < A$

3. If $2^a = 5$ and $2^b = 3$ then $\log_3 10 =$

   a) $\frac{a+1}{b}$ b) $\frac{b+1}{a}$ c) $\frac{b+1}{a+1}$ d) $\frac{a}{b}$ e) $\frac{b}{a}$

4. Find $f(5)$ if for all real numbers x, the function $f(x)$ satisfies

$$2f(x) + f(1 - x) = x^2.$$

   a) 4 b) 34/3 c) 8 d) 19/3 e) 12

5. The value of $\sin^2(10°) + \sin^2(20°) + \cdots + \sin^2(90°)$ is

   a) 5 b) 6 c) 4 d) 9/2 e) 21/4

6. A contest among $n \geq 2$ players is held over a period of 4 days. On each day each player receives a score of $1, 2, \ldots, n$ points with no two players getting the same score on a given day. At the end of the contest it is discovered that every player had the same total of 26 points. How many players participated?

   a) 8 b) 9 c) 10 d) 11 e) 12

7. What is the ratio of the area of a square that circumscribes a circle to the area of a square that inscribes the same circle?

a) $\sqrt{2}$ b) $\sqrt{3}$ c) 2 d) $\sqrt{6}$ e) 3

8. Let $P_n = 1^n + 2^n + 3^n + 4^n$. Find the number of integers n for which $1 \leq n \leq 100$ and P_n is a multiple of 5.

a) 68 b) 75 c) 86 d) 98 e) 100

9. Define a sequence by $b_1 = 2$ and

$$b_{n+1} = \frac{1 + b_n}{1 - b_n} \quad \text{for} \quad n \geq 1.$$

What is the value of b_{2013}?

a) 2 b) -3 c) -1/2 d) 1/3 e) -2013

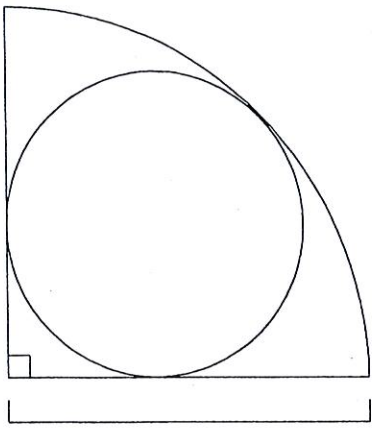
10. $\sin(\cos^{-1}(\tan(\pi/6))) =$

a) $\sqrt{2/3}$ b) $\sqrt{3}/2$ c) $\sqrt{2}/2$ d) $\pi/4$ e) 1/2

11. If the number 2013! were written in base 9, how many zeros would it end with?

a) 249 b) 334 c) 501 d) 1002 e) none of these

12. A circular sector has radius 1 and angle 90 degrees. A circle is inscribed in it, as shown below. What is the area of this circle?



a) $\pi(3 - 2\sqrt{2})$ b) $\frac{\pi}{4}$ c) $\frac{\pi(3-2\sqrt{2})}{2}$ d) $\frac{\pi}{8}$ e) answer not given

13. Agatha, Barnabus, and Corneilius are painting a fence together. They have timed themselves before and noted that Agatha and Barnabus working together can paint 70 feet of fence in an hour, Agatha and Corneilius can paint 110 feet of fence in an hour, and Barnabus and Corneilius can paint 60 feet of fence in an hour. If all three work together, how long will it take to paint a 120 foot fence?

a) 30 minutes b) 40 minutes c) 45 minutes d) 55 minutes e) 60 minutes

14. A bakery has 8 different kinds of donuts. In how many ways can a dozen donuts be selected?

a) $\binom{22}{12}$ b) $\binom{21}{12}$ c) $\binom{20}{12}$ d) $\binom{19}{12}$ e) None of these

15. A boat travels along a river heading from point A downstream to point B in 3 hours. The boat makes the return trip upstream in 4 hours. How long would it take to float down this river (unassisted by an engine or some other power source) from point A to point B?

a) 12 hours b) 18 hours c) 20 hours d) 24 hours e) 30 hours

16. What is the units digit of $1^2 + 2^2 + ... + 2013^2$?

a) 1 b) 4 c) 5 d) 6 e) 9

17. A driving instructor purchases new traffic cones when his old ones are cracked, dented, and losing their reflective coating. In his current stock, 65% of the cones are cracked, 80% are dented, and 70% have lost their coating. What is the smallest possible percentage of traffic cones in this bunch that will need to be replaced?

a) 15% b) 30% c) 20% d) 35% e) 65%

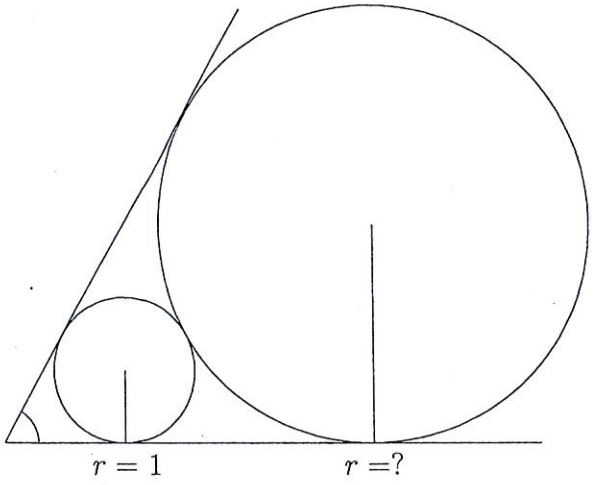
18. How many ways are there to select four of the numbers from the set $S = \{1, 2, \ldots, 10\}$ such that no two of them are consecutive?

a) 10 b) 25 c) 35 d) 40 e) 45

19. I have 11 Scrabble tiles, 7 of which are of the letter O and 4 of which are of the letter L. How many unique 11-letter "words" can I make with these tiles that are palindromes?

a) 5 b) 6 c) 10 d) 15 e) 24

20. A circle of radius 1 is inscribed between 2 rays that are 60° apart. A second larger circle is inscribed between these two rays such that it lies tangent to the first circle. (See diagram below.) What is the radius of the larger circle?



a) 2 b) 3/2 c) $\sqrt{3}$ d) 3 e) $2\sqrt{3}$

21. The value of $\log_{10} 2 = 0.301...$ is accurate to as many decimal places as shown. How many decimal digits are in the number 5^{80}?

a) 56 b) 57 c) 58 d) 59 e) 60

22. The numbers $1, 2, 3, \ldots, 9$ are placed into a 3×3 array so that each number occurs exactly once. The probability that the sum of the numbers in at least one horizontal row is greater than 21 is

a) 1/7 b) 1/9 c)1/10 d) 1/15 e) 1/21

23. There is one real root α of $4x^{100} - 2x^2 + 3x - 1$ satisfying $0.15 < \alpha < 0.65$. Which of the following is the nearest in value to α?

a) 0.2 b) 0.3 c) 0.4 d) 0.5 e) 0.6

24. Suppose that $f(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$ and that $f(1) = f(2) = f(3) = f(4) = f(5)$. Then $a =$

a) -8 b) 10 c) -15 d) 22 e) -35

25. Consider the following system of equations:

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$
$$3^{\ln x} = 2^{\ln y}$$

If the coordinate (x_0, y_0) is the solution to this system, then x_0 is:

a) 1/6 b) 1/3 c) 1/2 d) 2 e) 6